Sensorveiledning: MET 29104 Matematikk for økonomer

Eksamensdato: 27.05.2010, kl. 13.00 – 17.00

Tillatte hjelpemidler: Alle + eksamenskalkulator TEXAS INSTRUMENTS BA II Plus™

Totalt antall sider: 3

Oppgave 1

a) $f(x) = 3x^3 + x^{-3} - x \quad f'(x) = 3 \cdot 3x^2 - 3x^{-4} - 1 = \underline{\underline{9x^2 - 3x^{-4} - 1}}$

b) $f(x) = \frac{8}{x} - 11x \qquad f'(x) = \underline{\underline{-\frac{8}{x^2} - 11}}$

c) $f(x) = 4(x^2 + x)^{-2} \qquad f'(x) = 4 \cdot (-2)(x^2 + x)^{-3}(2x + 1) = \underline{\underline{-8(x^2 + x)^{-3}(2x + 1)}}$

d) $f(x) = 8xe^x \qquad f'(x) = 8e^x + 8xe^x = \underline{\underline{8e^x(1 + x)}}$

e) $\int (3x^2 + 2x^3 - 1)dx = 3 \cdot \frac{1}{3}x^3 + 2 \cdot \frac{1}{4}x^4 - x + C = \underline{\underline{x^3 + \frac{1}{2}x^4 - x + C}}$

f) $\int_1^3 (x + \frac{1}{x})dx = \left[\frac{1}{2}x^2 + \ln|x|\right]_1^3 = (\frac{1}{2} \cdot 3^2 + \ln 3) - (\frac{1}{2} \cdot 1^2 + \ln 1) = \frac{9}{2} + \ln 3 - \frac{1}{2} - 0 = \underline{\underline{4 + \ln 3}}$

g)

$f(x, y) = -x^2 + 3xy + y^2 - 12x - 34y + 136$

$F(x, y) = f(x, y) - \lambda(x + y - 8)$

$F(x, y) = -x^2 + 3xy + y^2 - 12x - 34y + 136 - \lambda x - \lambda y + 8\lambda$

$F'_x(x, y) = -2x + 3y - 12 - \lambda$

$F'_y(x, y) = 3x + 2y - 34 - \lambda$

$I : -2x + 3y - 12 - \lambda = 0$

$II : 3x + 2y - 34 - \lambda$

$III : x + y = 8$

$II - I : 5x - y - 22 = 0$

$III : x + y = 8$

$(II - I) + III : 6x = 30$

$x = 5$

$III : 5 + y = 8$

$y = 3$

$(x, y) = \underline{\underline{(5, 3)}}$

Oppgave 2

a)

$$6p = 120 - x(p) \qquad \underline{\underline{p = 20 - \frac{x(p)}{6}}}$$

$$I(x) = p \cdot x = (20 - \frac{x}{6})x = \underline{\underline{20x - \frac{x^2}{6}}}$$

b)

$$I'(x) = 20 - \frac{2x}{6} = 20 - \frac{x}{3}$$

$$I'(x) = 4$$

$$20 - \frac{x}{3} = 4$$

$$60 - x = 12$$

$$\underline{\underline{x = 48}}$$

c) $$E_p = x'(p)\frac{p}{x(p)} = -6\frac{p}{120 - 6p} = \underline{\underline{\frac{p}{p - 20}}}$$

d)

$$E_p < -1$$

$$\frac{p}{p - 20} < -1$$

$$\frac{p}{p - 20} + 1 < 0$$

$$\frac{p}{p - 20} + \frac{p - 20}{p - 20} < 0$$

$$\frac{2p - 20}{p - 20} < 0$$

0_____10_____20 *p-linje*

_ _ _ _ _ _ _ _ _ _ _ _0_____ *2p-20*

_ *p-20*

_____ 0 _ _ _ _ _ _ _ _ _ $\frac{2p - 20}{p - 20}$

$$E_p < -1 \Rightarrow \underline{\underline{10 < p < 20}}$$

Oppgave 3

a)

$$f(x) = x^4 - 2x^2 = x^2(x^2 - 2) = x^2(x + \sqrt{2})(x - \sqrt{2})$$

$$f(x) = 0 \Rightarrow \underline{\underline{x = 0, x = -\sqrt{2} \vee x = \sqrt{2}}}$$

_____ $-\sqrt{2}$ _____0_____ $\sqrt{2}$ _____ x-linje
_____0_____ x^2
_ _ _ _ _ 0_____ $x+\sqrt{2}$
_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ 0 _____ $x-\sqrt{2}$
_____ 0 _ _ _ _ _ 0 _ _ _ _ 0 _____ $f(x)$

$f(x) < 0 \Rightarrow \underline{\underline{-\sqrt{2} < x < 0 \vee 0 < x < \sqrt{2}}}$

$f(x) > 0 \Rightarrow \underline{\underline{x < -\sqrt{2} \vee x > \sqrt{2}}}$

b)

$f'(x) = \underline{\underline{4x^3 - 4x}}$

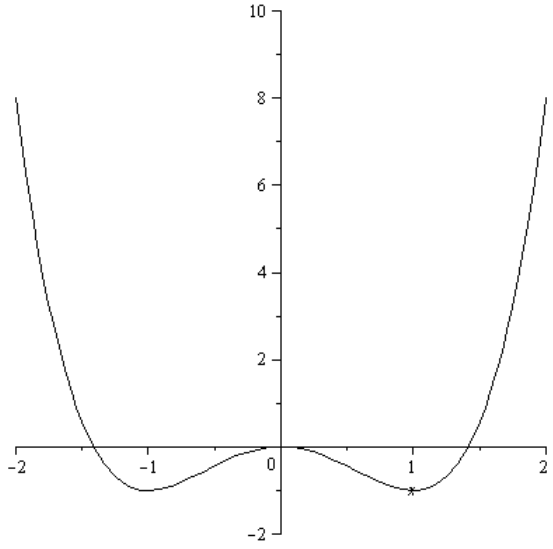
$f'(x) = 4x(x^2 - 1) = 4x(x+1)(x-1)$

_____ -1_____ 0_____ 1_____ x-linje
_ _ _ _ _ _ _ _ _ _ _0_____ $4x$
_ _ _ _ _ _ 0_____ $x+1$
_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _0 _____ $x-1$
_ _ _ _ _ _ 0 _____ 0 _ _ _ _ 0 _____ $f(x)$

$f(x)$ er voksende når $\underline{\underline{-1 < x < 0 \vee x > 1}}$

$f(x)$ er avtagende når $\underline{\underline{x < -1 \vee 0 < x < 1}}$

c)



d)

$g(x) = f(x) + c$

$g(x) = x^4 - 2x^2 + c \Rightarrow g(2) = 8 + c$

$g'(x) = 4x^3 - 4x \Rightarrow g'(2) = 24$

Tangent:

$y - g(2) = g'(2)(x - 2)$

$y - (8 + c) = 24(x - 2)$

$y - 8 - c = 24x - 48$

$y = 24x - 40 + c$

$x = 0, y = -10 \Rightarrow -10 = 24 \cdot 0 - 40 + c$

$\underline{\underline{c = 30}}$

Oppgave 4

$$K_0 = K\frac{(1+r)^n - 1}{(1+r)^n \cdot r}$$

i) $PV_1 = 25000\dfrac{1.034^{16} - 1}{1.034^{16} \cdot 0.034} = \underline{\underline{304636.16}}$

ii) $PV_2 = 55000 \cdot \dfrac{1}{1.034} \cdot \dfrac{1.034^7 - 1}{1.034^7 \cdot 0.034} = \underline{\underline{326458.94}}$

iii) $PV_3 = 280000 + 4000\dfrac{1.034^{11} - 1}{1.034^{11} \cdot 0.034} = \underline{\underline{316203.80}}$

Han bør velge alternativ 2

Oppgave 5

a)

$$A = \begin{bmatrix} 3 & 3 \\ 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 3 \\ 1 & -2 \end{vmatrix} = 3 \cdot (-2) - 3 \cdot 1 = -9$$

Siden determinanten til koeffisientmatrisen er forskjellig fra null har likningssystemet entydig løsning.

b)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 2 & -1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = -2 - 1 \cdot (-2 - 1) + 1 \cdot 4 = -2 + 3 + 4 = 5$$

$$|B_3| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = 0 - 1 \cdot 2 + 1 \cdot 4 = 2$$

$$z = \frac{|B_3|}{|A|} = \underline{\underline{\frac{2}{5}}}$$

Oppgave 6

$$f'(x) > 0 \Rightarrow \underline{\underline{1 < x < 5}}$$